## ※ATSE

$$
\begin{aligned}
& \text { gitlr } \\
& \text { PROJECT }
\end{aligned}
$$

## MATHEMATICS

## : Inl <br> Solar Panels



## CONTENTS

MATHEMATICS OF SOLAR PANELS ..... 3
InTRODUCTORY ACTIVITIES .....  4
THE SUN APPEARS TO MOVE .....  7
SEASONS AND LATITUDE ..... 27
R00F ANGLES ..... 33
SOLAR PANELS ON ROOFS ..... 42
Introduction ..... 42
OPEN ENDED INQUIRY ..... 58
GLOSSARY ..... 59

## MATHEMATICS OF SOLAR PANELS



## INTRODUCTION

These activities are designed for use with the STELR Renewable Energy module and equipment.
The STELR website and further information including contact details can be accessed at: www.stelr.org.au

## FOCUS QUESTIONS

- If you are putting solar panels on your roof, how do you position them to generate the most electricity?
- Why are solar panels usually mounted on an angle?
- Is the amount of electricity generated by solar panels affected by the seasons?
- Is the amount of electricity generated by solar panels affected by the time of day?
- Are solar panels more effective at different locations on Earth?


## INTRODUCTORY ACTIVITIES

## Activity 1: Terminology

During this STELR Solar Energy module, a variety of terms will be used. Some of these may be familiar to you, however it is encouraged that you have a detailed understanding of each and how each are different.

Note: When researching the internet for information, be aware that the information may refer to the Northern Hemisphere. Be sure to switch for southern conditions.

The Sun 'moves east to west' (same as Northern Hemisphere), facing north that is right to left...but facing south, east to west is left to right. As Australia and New Zealand are in the Southern Hemisphere, the Sun is north of our position, the Sun moves from right to left.

Your task: From notes or a dictionary, explain the meanings of each of the following terms.

| Angle of elevation <br> (altitude) |  |
| :--- | :--- |
| Angle of inclination <br> (pitch) |  |
| Earth's axis |  |
| Equinox |  |
| Gnomon |  |
| Horizontal |  |
| Latitude |  |
| Longitude |  |
| Meridian |  |
| Orientation |  |
| Solstice |  |
| Summer solstice |  |
| True bearing <br> (Azimuth) |  |
| Vertical |  |
| Winter solstice |  |
| Zenith |  |

Check with others in your class to make sure you agree on the definitions of each of the terms and that all of you understand what they mean.

## Activity 2: Inquiry questions

Before starting the investigation, make some predictions about each of the following:

How does the position of the Sun (angle of elevation and true bearing) change through the day?

## Angle of elevation (altitude):

$\qquad$
$\qquad$

True bearing (azimuth):
$\qquad$
$\qquad$
$\qquad$

At what time of the day is the Sun at its highest point (maximum elevation) in the sky?
$\qquad$

Which is 'moving'? The Earth or the Sun?
Why does the Sun appear to 'move'? How do you 'know' this?

Calculate how fast the 'Sun appears to be moving' in $\mathrm{km} / \mathrm{hr}$ and in $\mathrm{m} / \mathrm{s}$

## Necessary data:

Earth is an average 150 million km from the Sun.
The orbit radius ranges between 152 million km and 147 million km .
Calculate the circumference of a circular orbit using $2 \pi$ r where $\pi=3.14$
(Assume a circle with radius 150 million km )
The time for one orbit $\left(360^{\circ}\right)$ is one year or 365.25 days
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Activity3: Predicting how to position solar panels

Solar (photovoltaic or PV) panels are combinations of silicon 'cells' which generate electrical energy by conversion of light energy and are commonly found on rooftops which facing toward the Sun.


In this activity, you can predict the best orientation (direction and angle of inclination) for fixed solar panels where you live. In further activities, these predictions will be tested.

You can use Google Earth or Google Maps to help with this.

Find your house (or school) and zoom in.
Set the image so that North is at the top of the picture.

Make a quick outline sketch of the orientation of your
 house and then suggest which part of the roof would be best to put
 the panels on.

Give reasons for your choice of location and for which part of the roof:

## Risks

You should not look directly at the Sun. Give the reason why.

## THE SUN APPEARS TO MOVE

## Activity 1: The Sun at your location

## 'The Sun appears to move..'

The path of the Sun across the sky changes for each day of the year, and the difference between winter sunlight and summer sunlight can be observed.


Your task: List some of the similarities and differences between summer sunlight and winter sunlight.

## Similar

Always appears north of observer's location (assuming the observer is in the Southern Hemisphere and not between the Equator and the Tropic of Capricorn).
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Different

The position of sunrise and sunset changes
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Activity 2: The Sun path at different places

This activity is for those of you who want to learn more about the Sun appears to move through the sky for different places on Earth's surface.

## Background information

Because the Earth's axis of rotation is tilted, the position where the Sun seems to rise and set as well as the path across the sky changes during the year.

This also means the position of shadow at local noon will change from day to day.

If you go outside at local noon in the middle of summer, autumn, winter and spring:

Where will the Sun be?
Summer:
Winter:
Spring and Autumn:

## How long will your shadow be?

Summer:
Winter:
Spring and Autumn:

Hints: In the middle of spring and autumn, people standing on the Equator have the Sun directly overhead and their shadow is underneath them.

In July, this is true for people standing on the Tropic of Cancer in the Northern Hemisphere.
In December, it is true for people standing on the Tropic of Capricorn.

For people in Australia, Rockhampton and Alice Springs are almost on the Tropic of Capricorn. What can you conclude about the position of the Sun at different times of the year for people living south of these two towns?

Local noon is when the Sun reaches the highest point in the sky. Brisbane, Sydney, Canberra, Melbourne and Hobart are all on Eastern Standard Time, so 12:00 pm (clock noon) occurs at the same time in all of these cities.

What is the difference between 12:00 pm (clock noon) and local noon?

If you were at the same latitude north of the equator, where would you shadow be at local noon?

## Activity 3: Sun movement simulator

In a browser, view the following URL: [Accessed June 2016]
http://astro.unl.edu/naap/motion3/animations/sunmotions.html
The Motions of the Sun is a simulator that shows how time of year and observer's location will affect the position of the Sun in the sky.

Your controls are:
Day of the year
Time of day
Location (latitude) on Earth
An animation can show how the position of the Sun will change during the year.
You can alter the angle of the Earth by moving the green disk.(Grab and drag)


## Your task:

Compare different times of day and different times of year with chosen locations, record the altitude and azimuth.

Initial conditions: Set time of day to 12pm (noon)-default setting Compare and record altitude and azimuth (A) for each time of year(C) corresponding to each equinox and solstice. (eg: March 21, June 22, September 22, December 21)

Set the observer's latitude (B) by typing directly in the box or by sliding the indicator arrow beside the 2D map of Earth. Click on N/S to switch the value for the hemisphere.

Complete the following table after setting values for Latitude and time of year.

| Location | Latitude | March 21 <br> Equinox |  | June 22 <br> Solstice |  | September 22 Equinox |  | December 21 Solstice |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Altitude | Azimuth | Altitude | Azimuth | Altitude | Azimuth | Altitude | Azimuth |
| $1$ <br> Equator | $0^{\circ}$ |  |  |  |  |  |  |  |  |
| $2$ <br> Tropic of Cancer | $23.5{ }^{\circ} \mathrm{N}$ |  |  |  |  |  |  |  |  |
| 3 <br> Tropic of Capricorn | $23.5^{\circ} \mathrm{S}$ |  |  |  |  |  |  |  |  |
|  | $66.5^{\circ} \mathrm{N}$ |  |  |  |  |  |  |  |  |
| 5 <br> Antarctic Circle | $66.5^{\circ} \mathrm{S}$ |  |  |  |  |  |  |  |  |
| 6 <br> Your location |  |  |  |  |  |  |  |  |  |

What do you notice about the 'yellow Sun circle' (declination)?

What is significant about April 15, June 13, September 1 and December 25?

What do you notice about the equinox and solstice values for each location?

What do you notice about the altitude values for the tropics and for the polar circles?
Equinox the same, solstice reversed, the differences are close to $23.5^{\circ}$.

At the equinoxes, what do you notice about the azimuth values?

Calculate the difference in altitude between a successive equinox and solstice.

| Location | Latitude | Altitude |  | Difference | Altitude |  | Difference |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | March <br> 21 | June 22 |  | June <br> 22 | Sept $22$ |  |
| 1 <br> Equator | $0^{\circ}$ |  |  |  |  |  |  |
|  | $23.5{ }^{\circ} \mathrm{N}$ |  |  |  |  |  |  |
| 3 <br> Tropic of Capricorn | $23.5{ }^{\circ} \mathrm{S}$ |  |  |  |  |  |  |
| 4 <br> Arctic Circle | $66.5^{\circ} \mathrm{N}$ |  |  |  |  |  |  |
| 5 <br> Antarctic Circle | $66.5^{\circ} \mathrm{S}$ |  |  |  |  |  |  |
| 6 <br> Your location |  |  |  |  |  |  |  |

Activity 4: The apparent path of the sun at different places
Match the following sun position diagrams with the position on Earth
Your location choices are:

1) Northern Hemisphere
2) Southern Hemisphere
3) Equator
4) Tropic of Capricorn
5) Tropic of Cancer
6) Arctic Circle
7) Antarctic Circle

## Activity 5: Sun bearings

## Background information

The position of the Sun changes through the day as the Earth rotates on its axis. It rises in the East each morning, and sets in the west every night.

The path that the Sun appears to follow across the sky also changes throughout the year. This is caused by the tilt of the Earth on its axis.

In winter, the Sun is lower in the sky (in the Southern Hemisphere) while in summer it is higher.

Your distance north or south of the Equator (latitude) determines exactly how high the Sun will reach.


The diagram below is a compass laid flat on the ground with $0^{\circ}$ pointing North.

Label the lines that have the following bearings:

A 092 degrees
B 243 degrees
C 117 degrees
D 291 degrees
N 000 degrees

Rule in the 'ordinal' points of the compass
E east
W west
S south
N north


## Activity 6: Sun bearings and solar panels

## You will need:

- 30 cm ruler
- Bearing Alignment Circle page
- Magnetic compass (either a physical one or a phone app are fine. Be careful not to stand near metal or electrical wires when doing this activity as both can pull the compass needle away from the correct position of North.)
- STELR solar panel with multimeter


## Your task:

1) Place the Bearing Alignment Circle on top of a book or a folder so that it rests horizontally.
2) Place the compass on top of the Alignment Circle.
3) Turn the Alignment Circle until North on the paper is the same as the needle on the compass. It should look like this:
4) Check with your teacher if you need to make a further
 adjustment for true North. This depends on local magnetic field. (Teacher see pages 12 and 13)
5) Now take the compass away and place the ruler In the middle of the circle standing on one end.
6) Turn the ruler about its vertical axis until the edges of the shadow are at right angles to the ruler. You have just lined up the bearing of the Sun. You can read the bearing by looking at the angle value directly opposite the shadow. Practice using the following diagram.

The bearing of the example is $40^{\circ}$
7) Now record your actual result

The bearing of the Sun is $\qquad$ 0

8) Now you are ready to line up the STELR solar panel with the Sun.

Solar panels, like the one in the photograph, can be connected to motors, light bulbs or meters. Depending on how much light is falling onto the solar panel, more or less electricity will be generated. The maximum electricity generation will occur when the panel is pointing directly ay the Sun. The same is true for solar panels on buildings.

Connect the four solar cells in on the STELR solar panel in series at the back using the yellow wires

Then connect the STELR solar panel to the multimeter as shown in the picture.

Take your solar cell, ruler and Alignment Circle and set them up as in the diagram.

Lean your solar cell against the ruler and change the angle until the voltmeter reaches the maximum reading. Note the maximum voltage: $\qquad$

The angle you have found is called the pitch angle.
9) Calculating the pitch angle.
a. Measure the height of the solar cell from the ruler. Remember to allow for the bottom part of the ruler where there are no markings.
b. Measure the horizontal distance from the base of the ruler to the solar panel.
c. Now use trigonometry to calculate the pitch angle.

Pitch angle $=$ _


Wiring at back of STELR solar panel


STELR solar panel connected to multimeter

10)The angle of elevation for the solar panel
$=\left(90^{\circ}-\right.$ pitch angle $)$
Angle of elevation $=$ $\qquad$。

Challenge: Prove the formula for angle of elevation given in 10) above.

Record your results in the spaces provided
Date:
Time:
$\qquad$

Bearing:
$\qquad$

Angle of elevation:
$\qquad$
$\qquad$

## Discussion

How did you make sure the ruler was vertical?

How can you use these results to find the best orientation (direction and angle of slope) for fixed solar panels where you live?

Describe how the 'best' angle of slope is likely to change depending on:
The time of year.

Your distance from the Equator.

In this exercise, you used a small solar cell and moved it up and down to align it properly.
Give two reasons as to why this is not a feasible approach when installing full size solar panels on the roofs of houses and other buildings.
$\qquad$
$\qquad$

Look at the prediction that you made regarding positioning solar panels in the roof in the Activity: Predicting how to position solar panels. In light of what you have found out, would you now alter your prediction? Explain your answer.

## Activity 7: Observing ‘local noon’ and calculating True North

In this activity, you will:

- verify the Sun moves at $1^{\circ}$ each 4 minutes and
- measure the azimuth or altitude angle of the Sun at your local noon.

Your calculations will indicate True North from your position.

## You will need:

- Large protractor, metre ruler (x 2) and retort stand
- Chalk, pencil and paper or a method of recording the Sun shadow (eg. camera)
- Timer or clock
- Magnetic compass
- North facing open space with a 'flat' surface.

Your task: (A sunny day is recommended for this activity)
Ideally, you will observe local noon, your teacher will arrange for you to measure the Sun shadow either side of the moment your local noon occurs.11:45am to 12:30pm local time is usually adequate.
(Note: Daylight saving time will adjust this to $12: 45 \mathrm{pm}$ to $1: 30 \mathrm{pm}$ ).

1) Place the metre ruler vertically, on a flat (horizontal) surface.
2) Use the retort stand as support.
3) The metre ruler is acting as a gnomon for this sundial
4) Record the length of the shadow and the time of the observation
5) Repeat the observation every 5 minutes for half an hour, longer if possible.

Results table 1 - Shadow length and bearing

| Time of day | Shadow length | Bearing (from true North) |
| :--- | :--- | :--- |
| $11: 45 \mathrm{am}$ |  |  |
| $11: 50 \mathrm{am}$ |  |  |
| $11: 55 \mathrm{am}$ |  |  |
| $12: 00 \mathrm{pm}$ |  |  |
| $12: 05 \mathrm{pm}$ |  |  |
| $12: 10 \mathrm{pm}$ |  |  |
| $12: 15 \mathrm{pm}$ |  |  |
| $12: 20 \mathrm{pm}$ |  |  |
| $12: 25 \mathrm{pm}$ |  |  |
| $12: 30 \mathrm{pm}$ |  |  |

## Analysis of your data:

If the time of day has been carefully chosen, you will notice the shadow gets shorter and then gets longer.

Draw a graph of your results plotting Time vs. Shadow length.
Determine the time of shortest shadow (interpolation may be required), this is the time of local noon.
The direction of True North is the shadow of shortest length and this shadow occurs at local noon.
ie. The Sun is due North at local noon

Did the shortest shadow occur before or after 12:00noon clock time?

What is the angle difference between time of shortest shadow and clock noon?
$\qquad$

Why does the shortest shadow occur at local noon, and not time zone noon?

Further calculations can be performed on your data to give the elevation of the Sun.


Use trigonometry to calculate the Sun height angle. (Hint: Use tangent)

Enter the data into results table 2.

Results table 2 - Sun elevation

| Time of day | Sun elevation angle | Bearing (from true North) |
| :--- | :--- | :--- |
| $11: 45 \mathrm{am}$ |  |  |
| $11: 50 \mathrm{am}$ |  |  |
| $11: 55 \mathrm{am}$ |  |  |
| $12: 00 \mathrm{pm}$ |  |  |
| $12: 05 \mathrm{pm}$ |  |  |
| $12: 10 \mathrm{pm}$ |  |  |
| $12: 15 \mathrm{pm}$ |  |  |
| $12: 20 \mathrm{pm}$ |  |  |
| $12: 25 \mathrm{pm}$ |  |  |
| $12: 30 \mathrm{pm}$ |  |  |

Draw a graph of your results plotting Time vs. Sun elevation angle.
Determine the time of greatest altitude (Interpolation may be required), this is the time of local noon.

Calculate the difference between local noon and clock time noon
If one hour is equivalent to $15^{\circ}$, (or 4 minutes per $1^{\circ}$ ) use the angle difference between local noon and time zone (or clock time) noon to calculate an estimate for the time difference between local noon and time zone noon.

Calculate the difference between your time zone noontime and your local noon.

Compare your results with an equation of time calculator.
You can locate the Solar (local) noon calculator at this URL:
http://www.solar-noon.com/sn form.html
[URL accessed June 2016]
How do they compare?

How closely do they agree? Explain why you think they are the same, or different.

## Activity 8: local noon and time zone noon differences - method 1

Local noon is the time when the Sun is highest in the sky and it depends on your longitude. The 'clock time' is determined by time zones, which are based on noon at a chosen longitude.
For example: Brisbane, Hobart, Melbourne and Sydney are all in the same time zone, Australian Eastern Standard Time. This time zone is based on longitude $150^{\circ}$ meridian.

## Your task

Use the program http://www.wolframalpha.com to find the maximum altitude time at different locations. In the search box type: " sunpath <your location>". For example if you want to find the sunpath for Melbourne, type " sunpath melbourne, victoria"

Find the next maximum altitude time.
Maximum altitude time is the clock time for local noon.

Calculate the differences in longitude between cities and towns in these Australian time zones The Sun takes four minutes to travel one degree of longitude.

Note that:

- if local noon occurs before clock noon then the longitude difference is added to the time zone longitude
- if local noon occurs after clock noon then the longitude difference is subtracted from the time zone longitude.
- 

| City or town | Time zone <br> longitude | Clock time for <br> local noon | Local noon <br> correction <br> (minutes) | Longitude <br> difference <br> $\left({ }^{\circ}\right)$ | Longitudle |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | AEST |  |  |  |  |
| Brisbane | $150^{\circ}$ |  |  |  |  |
| Hobart |  |  |  |  |  |
| Melbourne |  |  |  |  |  |
| Mildura |  |  |  |  |  |
| Mt Isa |  |  |  |  |  |
| Lord Howe Is |  |  |  |  |  |
| Sydney |  |  |  |  |  |
|  | ACST |  |  |  |  |
| Adelaide | $142.5^{\circ}$ |  |  |  |  |
| Alice Springs |  |  |  |  |  |
| Ceduna |  |  |  |  |  |
| Broken Hill |  |  |  |  |  |
| Darwin |  |  |  |  |  |
| Kangaroo Is |  |  |  |  |  |
| Renmark |  |  |  |  |  |
|  |  |  |  |  |  |
| Albany | $120^{\circ}$ |  |  |  |  |
| Broome |  |  |  |  |  |
| Kalgoorlie |  |  |  |  |  |
| Perth |  |  |  |  |  |
| Port Hedland |  |  |  |  |  |

Please note that the data downloaded into column 3 is date-dependent.

## Activity 9: Local noon and time zone noon differences - method 2

Local noon is the time when the Sun is highest in the sky and it depends on your longitude. The 'clock time' is determined by time zones, which are based on noon at a chosen longitude. It takes four minutes for the Sun to travel one degree of longitude.

For example: Brisbane, Hobart, Melbourne, Mildura and Sydney are all in the same time zone, Australian Eastern Standard Time. This time zone is based on the meridian with longitude $150^{\circ}$. Calculate the differences between time zone noon and local noon for cities and towns in these Australian time zones using the formula:
Local noon correction $=$ (Time zone longitude - Longitude) $\times 4$ mins Longitudes for different places can be found here: [URL accessed June 2016] http://www.heavens-above.com/SelectLocation.aspx?lat=0\&Ing=0\&loc=Unspecified\&alt=0\&tz=UCT

| City or town | Longitude | Time zone <br> longitudle | Longitude <br> difference <br> $\left({ }^{\circ}\right)$ | Local noon <br> correction <br> (minutes) | Clock time <br> for local <br> noon |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | AEST |  |  |  |
| Brisbane |  | $150^{\circ}$ |  |  |  |
| Canberra |  |  |  |  |  |
| Hobart |  |  |  |  |  |
| Melbourne |  |  |  |  |  |
| Mildura |  |  |  |  |  |
| Mt Isa |  |  |  |  |  |
| Lord Howe Is |  |  |  |  |  |
| Sydney |  |  |  |  |  |
|  |  |  |  |  |  |
| Adelaide |  |  |  |  |  |
| Alice Springs |  |  |  |  |  |
| Ceduna |  |  |  |  |  |
| Broken Hill |  |  |  |  |  |
| Darwin |  |  |  |  |  |
| Kangaroo Is |  |  |  |  |  |
| Renmark |  |  |  |  |  |
|  |  |  |  |  |  |
| Albany |  |  |  |  |  |
| Broome |  |  |  |  |  |
| Kalgoorlie |  |  |  |  |  |


| Perth |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Pt Hedland |  |  |  |  |  |
|  |  | ACWST |  |  |  |
| Mundrabilla |  | $127.5^{\circ}$ |  |  |  |

Which location has the greatest time zone correction? $\qquad$

## SEASONS AND LATITUDE

## Activity 1: Solar astronomy of your local seasons

Find your local latitude and longitude. You will use these measurements for local calculations. A GPS or an online map will provide a measurement accurate to the nearest 10 metres. You may choose a less precise measurement which includes you school and home.(e.g. five kilometres)

In a browser, use Wolfram Alpha (URL: http://www.wolframalpha.com) enter the search:
" sunpath <your location> winter solstice "
" sunpath <your location> summer solstice"
" sunpath <your location> equinox "

| Place: |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Longitude: |  |  | Angle of <br> elevation | Sunrise <br> bearing | Sunset <br> bearing |
| Solar event | Date | Length of <br> daylight hours |  |  |  |
| Summer <br> Solstice |  |  |  |  |  |
| Autumn <br> Equinox |  |  |  |  |  |
| Winter <br> Solstice |  |  |  |  |  |
| Spring <br> Equinox |  |  |  |  |  |

At what time of the year does the Sunrise and set due East and due West?
$\qquad$

What is the difference in daylight hours between the Summer solstice and the Winter solstice?
$\qquad$

What is the difference between the angle of elevation of the Sun at Summer solstice and equinox?

What is the difference between the angle of elevation of the Sun at Winter solstice and equinox?

What does this angle correspond to?

What is the relationship between latitude and the maximum angle of elevation of the Sun at equinox?
$\qquad$
$\qquad$

Complete the diagram below using your local conditions.

$\qquad$
Latitude angle $\mathrm{a}=$ 0

Tilt of the Earth $\varepsilon=$ $\circ$

Complete the following for your location:

| Time of year | Symbol | Equation | Angle of elevation <br> at my location |
| :--- | :--- | :--- | :--- |
| Summer solstice | $\delta$ | Equinox +23.4 |  |
| Equinox | $\gamma$ | $90-$ latitude |  |
| Winter solstice | $\beta$ | Equinox -23.4 |  |
| Equinox | $\gamma$ | 90 - latitude |  |



On the diagram above, mark in the solstice and equinox angles

How might the difference in the angle of altitude of the Sun in winter and summer determine how you position solar panels on your roof?


On the protractor above. Mark in:
a) Sunrise and sunset bearings for the Summer solstice.

Calculate the number of degrees difference between summer sunrise and sunset.
b) Sunrise and sunset bearings for the Equinox.

Calculate the number of degrees difference between equinox sunrise and sunset.
c) Sunrise and sunset bearings for the Winter solstice.

Calculate the number of degrees difference between winter sunrise and sunset.

How is difference between sunrise and sunset bearings related to the length of daylight hours?

How might the difference in the sunrise and sunset bearings in winter and summer determine how you position solar panels on your roof?

## Activity 2: What is a season?

Research the start dates of the seasons for the following countries:

## Northern Hemisphere

|  | Denmark | France | USA |
| :--- | :--- | :--- | :--- |
| Start of Spring |  |  |  |
| Start of Summer |  |  |  |
| Start of Autumn |  |  |  |
| Start of Winter |  |  |  |

## Southern Hemisphere

|  | Australia | South Africa | New Zealand |
| :--- | :--- | :--- | :--- |
| Start of Spring |  |  |  |
| Start of Summer |  |  |  |
| Start of Autumn |  |  |  |
| Start of Winter |  |  |  |

Some countries start their season dates on the equinoxes and the solstices. In Australia, the seasons begin on 1 December, 1 March, 1 June and 1 September. Why?

In areas close to the Equator, there are not four seasons. Look at the diagram below and use them to explain why the idea of summer and winter is meaningless close to the equator:


Australian indigenous people have several more seasons in different parts of the country.
There are five named and identified seasons in the Gulf country, six in the Broome area and the Walabunba people of Central Australia have three.
Research the seasons of an indigenous tribe including the name of the season, what it means and when it is.

## Extension Activity 3: angle of elevation of the Sun at the equinox

Show the relationship between angle of elevation of the Sun at the equinox and the observer's latitude Hint: Use opposite angles, angles in a triangle and alternate angles

Step 1: On the diagram below, mark your location on the surface of the Earth (edge of the circle)

Step 2: Identify your angle to the centre of the Earth (latitude), the horizon tangent and angle to the Sun to the horizon (angle of elevation).

Hint: Use angles in a triangle, alternate angles, opposite angles to show the angle of elevation of the Sun at the observer's latitude.

Step 3: Work through the known values to show the relationship between the angle of elevation of the Sun and the observer's latitude.

## ROOF ANGLES



## INTRODUCTION

These activities are designed for use with the STELR Renewable Energy module and equipment.
The STELR website and further information including contact details can be accessed at:
www.stelr.org.au

## FOCUS QUESTIONS

- If you are putting solar panels on your roof, how do you position them to generate the most electricity?
- Why are solar panels usually mounted on an angle?
- Is the amount of electricity generated by solar panels affected by the seasons?
- Is the amount of electricity generated by solar panels affected by the time of day?
- Are solar panels more effective at different locations on Earth?


## INQUIRY QUESTION

What is the angle that your chosen roof makes to the horizontal?

## RISK ASSESSMENT

It is too dangerous to climb onto a roof to measure the angle of the roof. You could fall off .
Can you think of any other risks?

## Method A - Measuring the angle using an inclinometer

An inclinometer is an instrument that measures the angle a line makes with the horizontal.

## You will need:

- Card (stiff)
- Glue
- Scissors
- A drinking straw
- Butterfly pin
- String or a piece of thin thread
- A weight such as a blob of Blu-tack or a metal nut
- Sticky tape
- Inclinometer template


## Your task:

Work in pairs.
Make the STELR inclinometer:

- Print the Inclinometer template.
- Glue the template to a piece of stiff card.
- Poke a hole where indicated and attach a piece of string using the butterfly pin.
- Attach a weight to the end of the string.
- Glue or tape the straw where indicated.

Measure the roof pitch.

- Stand in front of the building so you can


Butterfly pins see line of the roof.


- Look through the straw at the place where you see highest part of the roofline and the lower roofline.
- You partner can read off the angle where the string crosses the scale. Record the angle.
- Swap roles.

Callculate the average of your readings.
This is the angle your roof makes with the horizontal.

## Result

What is the angle of the roof to the horizontal? $\qquad$

## Method B - Measuring angle from a photograph

Use a camera to record the angle, then measure the angle using a protractor.
This method can be used if you are able to photograph the roof 'end-on' to get an undistorted view of the roof. You can also calculate the height of the roof.

Using the camera on your phone or tablet, take a photo of a house where you can see the angle of the roof.

Have a partner stand in the photo as well holding a metre ruler or tape measure set at 1 metre. This will be used to work out the scale of the photo to real life size. Send the photo to a computer so that you can print it.

Once you have printed the photo measure the length of the 1 metre ruler/tape measure.

Length in photo in $\mathrm{cm}=1 \mathrm{~m}$ in real life $\mathrm{mm}=1 \mathrm{~m}$ in real life
Now measure the roof height on the photo.
Convert to the actual height using proportions.

Measuring the roof angle
Use a protractor to measure the angle of the roof.


Now swap with your partner and let them make measurement of the roof angle.
First measurement $=$ $\qquad$ -

Second measurement $=$ $\qquad$ -

Average roof angle = $\qquad$。

## Method C - Using an angle measuring App

Download a free inclinometer apps for your smartphone or tablet. Phones are usually better to use for this purpose because you can hold them in one hand easily without fear of dropping them.
Working with a partner, have one person stand to the side of the house so that they can just sight along the roof. Hold the
 phone at arm's length and point your arm at the angle of the roof line a shown in the photo.

You need to line the phone and the roof up so that the top edge of the phone and the entire length of the roof line up.


The second person now reads the angle measured by the inclinometer app.

What amount of accuracy should you take for your measurement? The nearest degree? The nearest tenth of a degree? Somewhere in between? Give a reason for
 your answer.
$\qquad$
$\qquad$
$\qquad$

Now swap with your partner and take another measurement of the roof angle.
First measurement $=$ $\qquad$。

Second measurement $=$ $\qquad$ $\circ$

Average roof angle $=$ $\qquad$。

## Additional activity: calculating the roof height and the roof length

If you measure a base line, you can use trigonometry to calculate the roof height and the roof length. Using a tape measure, find the distance from the edge of the roof to the point directly under the peak. You will again need to work in pairs, with each person holding an end of the tape measure.


Describe how you determined where the two ends of the tape measure needed to be placed for greatest accuracy.

Swap ends and re-measure the length.

First length = $\qquad$

Second length = $\qquad$
If you had two different measurements, explain why.

Average base line length = $\qquad$

Now that you have one angle and one length measurement it is possible to calculate the other two sides of the triangle using trigonometry. On the diagram below, write down the base length and angle values that you measured earlier.

> Base line length - adjacent
> Roof height = opposite
> Roof length - hypotenuse


Calculate the height of the roof using the tangent ratio $\left(\tan \theta=\frac{o p p}{a d j}\right)$. Show your working.

Now calculate the length of the roof using the sine ratio $\left(\sin \theta=\frac{o p p}{h y p}\right)$. Show your working.

Enter the calculated values on the diagram.

## Activity: A solar panel on your roof

Chose a roof (for example - your house or your school).
Azimuth
You can use Google Earth or Google Maps to zoom in on your chosen building.
Set the image so that North is at the top of the picture.
Make a quick outline sketch of the orientation of the building.
An example for a roof in Melbourne has been completed as a guide.


Your building:

Use one of the methods in the previous activities to determine the roof angle (also known as the elevation)

For your chosen roof, fill in the azimuth and elevation.

| Horizontal (Bearing/Azimuth) |  |
| :--- | :--- |
| Vertical (Elevation) |  |

Now suggest which part of the roof would be best to put the panels on. Explain why.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Once the angles have been measured, other factors which need to be considered nearby local features which will block 'your' sunlight.

List some nearby local features which may block sunlight.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

It is possible to calculate how much reduction occurs and to compensate by changing the angle of the solar panel to gather more sunlight later or earlier in the day.
Which do you think is the worst direction for an obstruction for a solar panel?
Explain your reasoning.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Many newer panels can be connected together and some point east, some point west to avoid overshadowing from neighbouring obstructions.

Which direction has the least sunlight available, at all times of the year?

## SOLAR PANELS ON ROOFS



## INTRODUCTION

In these activities you will use simulation programs and mathematics to predict how the amount of electricity generated by solar panels is affected by their angle to the Sun.

You then test your prediction using STELR solar panels to generate electricity.

## FOCUS QUESTIONS

- If you are putting solar panels on your roof, how do you position them to generate the most electricity?
- Why are solar panels usually mounted on an angle?
- Is the amount of electricity generated by solar panels affected by the angle to the Sun?


## ACTIVITY 1: VISIBLE AREA - VERTICAL ROTATION USING A SIMULATOR

When you change your observation position the apparent shape of the object changes.

The rate of distortion, or dilation can be calculated and used to predict (or model) expected behaviour of a solar panel at different angles to the Sun.

You will need:
STELR solar panel.


## Your task:

Complete the following table:
Calculate the area of the STELR solar panel. Just measure the length and width of the four solar cells not the entire panel. (All measurements in cm )

| Length $(\mathrm{cm})$ |  |
| :--- | :--- |
| Width $(\mathrm{cm})$ |  |
| Area $\left(\mathrm{cm}^{2}\right)$ |  |

Now use the simulation below to work out the apparent area of the panel at different angles rotating about a vertical axis.
https://www.youtube.com/watch?v=m07wfRqO2Pc

## Angle changes $5^{\circ}$ per second



The apparent area changes as the angle changes.

The animation provides the percentage change in apparent area.

Copy the percentage area into the table below and then calculate the apparent area.

| Angle | Apparent area (\%) | Apparent Area $\left(\mathrm{cm}^{2}\right)$ |
| :--- | :--- | :--- |
| $0^{\circ}$ |  |  |
| $10^{\circ}$ |  |  |
| $20^{\circ}$ |  |  |
| $30^{\circ}$ |  |  |
| $40^{\circ}$ |  |  |
| $50^{\circ}$ |  |  |
| $60^{\circ}$ |  |  |
| $70^{\circ}$ |  |  |
| $80^{\circ}$ |  |  |
| $90^{\circ}$ |  |  |

Now plot the apparent area of the panel against the angle of rotation.


## ACTIVITY 2: VISIBLE AREA - VERTICAL ROTATION USING CALCULATIONS

In this activity trigonometry is used to calculate apparent area.
Calculate apparent area for this solar panel which is 1 metre wide and 2 metres tall.

As it rotates around the vertical axis the width of the panel will appear to change and the height will stay the same.


Use trigonometry to calculate the apparent width (aw) of the panel when it is rotated $30^{\circ}, 45^{\circ}$ and $60^{\circ}$.

aw

aw


Using the measurements of the STELR solar panel and the same method as above, calculate its apparent area for rotations from zero through to 90 degrees.

| Angle | Apparent width $(\mathrm{cm})$ | Height $(\mathrm{cm})$ | Apparent Area $\left(\mathrm{cm}^{2}\right)$ |
| :--- | :--- | :--- | :--- |
| $0^{\circ}$ |  |  |  |
| $10^{\circ}$ |  |  |  |
| $20^{\circ}$ |  |  |  |
| $30^{\circ}$ |  |  |  |
| $40^{\circ}$ |  |  |  |
| $50^{\circ}$ |  |  |  |
| $60^{\circ}$ |  |  |  |
| $70^{\circ}$ |  |  |  |
| $80^{\circ}$ |  |  |  |
| $90^{\circ}$ |  |  |  |

What is the relationship between the apparent area and the tilt of the panel?

## ACTIVITY 3: VISIBLE AREA - HORIZONTAL ROTATION USING A SIMULATOR

When you change your observation position the apparent shape of the object changes.

The rate of distortion, or dilation can be calculated and used to predict (or model) expected behaviour of a solar panel at different angles to the Sun.

You will need:

- STELR solar panel.


## Your task:



- Complete the following table:
- Calculate the area of the STELR solar panel. Just measure the length and width of the four solar cells - not the entire panel. (All measurements in cm)

| Length $(\mathrm{cm})$ |  |
| :--- | :--- |
| Width $(\mathrm{cm})$ |  |
| Area $\left(\mathrm{cm}^{2}\right)$ |  |

One way to visualise this change is to observe the shadow cast by the panel at the angle changes.
https://www.youtube.com/watch?v=Mhk5QlaAULo


The apparent area changes as the angle changes.

The animation provides the apparent percentage change in area.

Copy the percentage area into the table below and then calculate the apparent area.

| Angle | Apparent area (\%) | Apparent Area (cm ${ }^{\mathbf{2}}$ ) |
| :--- | :--- | :--- |
| $0^{\circ}$ |  |  |
| $10^{\circ}$ |  |  |
| $20^{\circ}$ |  |  |
| $30^{\circ}$ |  |  |
| $40^{\circ}$ |  |  |
| $50^{\circ}$ |  |  |
| $60^{\circ}$ |  |  |
| $70^{\circ}$ |  |  |
| $80^{\circ}$ |  |  |
| $90^{\circ}$ |  |  |

Now plot the apparent area of the panel against the angle of rotation.


## ACTIVITY 4: VISIBLE AREA - HORIZONTAL ROTATION USING CALCULATIONS

In this activity trigonometry is used to calculate apparent area.
Calculate apparent area for this solar panel that is one metre wide and two metres tall.
It will tilt back through 90 degrees until it is horizontal. At 90 degrees, the front face of the panel cannot be seen.

As it rotates around the horizontal axis, the height of the panel will appear to change and the width will stay the same.

1.00 m


Use trigonometry to calculate the apparent height (ah) of the panel when it is rotated $30^{\circ}, 45^{\circ}$ and $60^{\circ}$. The diagrams below are showing the panel (hypotenuse) and the viewer from the side.


Using the measurements of the STELR solar panel and the same method as above, calculate its apparent area for rotations from zero through to 90 degrees.

| Angle | Apparent height <br> $(\mathbf{c m})$ | width (cm) | Apparent Area (cm ${ }^{2}$ ) |
| :--- | :--- | :--- | :--- |
| $0^{\circ}$ |  |  |  |
| $10^{\circ}$ |  |  |  |
| $20^{\circ}$ |  |  |  |
| $30^{\circ}$ |  |  |  |
| $40^{\circ}$ |  |  |  |
| $50^{\circ}$ |  |  |  |
| $60^{\circ}$ |  |  |  |
| $70^{\circ}$ |  |  |  |
| $80^{\circ}$ |  |  |  |
| $90^{\circ}$ |  |  |  |

What is the relationship between the apparent area and the tilt of the panel?
$\qquad$
$\qquad$
$\qquad$

## ACTIVITY 5: OBSERVING THE RELATIONSHIP BETWEEN ANGLE OF ELEVATION AND VOLTAGE GENERATED

Usually a solar panel is permanently fixed at one angle to simplify construction and maintenance. Tracking solar panels are possible, but cost more to operate.

We have observed the apparent area changes when the angle of the solar panel changes. The amount of electricity generated by the solar panel is directly related to the amount of light incident on the panel.

In this activity, you will investigate the amount of electricity generated when the angle of incidence is changed.

Note: This experiment can be performed horizontally or vertically.
Horizontally may be simpler to setup and manage.

## You will need:

- STELR solar panel
- STELR light source and power pack
- (Note: STELR light source has enhanced output halogen globe which closely simulates the solar spectrum)
- STELR multimeter (set to 20 V scale)
- STELR cables
- Solar cell alignment circle (see page 15 )
- Marking tape, ruler or pen

Your task:
Connect the four solar cells on the STELR solar panel to each other in series at the back of the panel.


Series connection drawn from the front of the panel


Connect the panel to the multimeter


Series connection shown at the back of the panel


Connect the STELR lamp to the power pack and the power pack to the mains.

Determine the optimum distance to place the panel from the light source.
There will be a light beam 'spread' as you move the panel away from the source.
There will also be a light beam intensity decrease as you move the panel away from the source.

Choose a position for the panel where the light beam strikes the panel at an angle of $90^{\circ}$ and is centrally located for maximum voltage.
Test this by moving the panel towards and away from the panel until the voltage is a maximum, then move the panel left and right until the voltage is a maximum.

Mark this position on the bench with tape so the panel can be simply repositioned if moved or disturbed.


Solar panel pointing towards light source.
It is centred and at a distance which generates maximum voltage.

Measure and record the angle of elevation and record the voltage generated at that angle. All other factors remain the same.eg. Distance and other angles should remain constant.


Alter the angle by $10^{\circ}$ repeat the observation of angle and voltage.

There are ten observation points.

Would the observations be symmetrical for left and right?

| Angle | Solar panel voltage <br> (Light source ON) | Ambient voltage <br> (Light source OFF) | Solar panel <br> voltage |
| :--- | :--- | :--- | :--- |
| $0^{\circ}$ |  |  |  |
| $10^{\circ}$ |  |  |  |
| $20^{\circ}$ |  |  |  |
| $30^{\circ}$ |  |  |  |
| $40^{\circ}$ |  |  |  |
| $50^{\circ}$ |  |  |  |
| $60^{\circ}$ |  |  |  |
| $70^{\circ}$ |  |  |  |
| $80^{\circ}$ |  |  |  |
| $90^{\circ}$ |  |  |  |

Create a scatter plot and graph your results of voltage against angle.


Describe the relationship after graphing the results. A line of 'best fit' may be appropriate.

Compare your scatter plot with the graph you made of apparent area vangle of rotation. Comment on whether the amount of electricity generated by the solar panel at different angles was as predicted by the mathematical calculations.

## Error analysis

Estimate any errors involved.
How accurate and precise is the angle measurement?

| Angle |  |
| :--- | :--- |
| Precision |  |
| Accuracy |  |
| Total error |  |
| Percentage error on the largest reading |  |
| Percentage error on the smallest reading |  |

How accurate and precise are the voltage readings?

| Voltage |  |
| :--- | :--- |
| Precision |  |
| Accuracy |  |
| Total error |  |
| Percentage error on the largest reading |  |
| Percentage error on the smallest reading |  |

Total error is the sum of the relative or percentage errors.
What is the largest total percentage error?

What is the smallest total percentage error?

When does the largest error occur, at large or smaller angles? Explain why this happens.

## ACTIVITY 6: A SOLAR PANEL ON YOUR ROOF

For your chosen roof, determine the azimuth and elevation.

| Address |  |
| :--- | :--- |
| Horizontal (Bearing/Azimuth) |  |
| Vertical (Elevation) |  |

Magnetic variation correction: True North may be different to magnetic north. Use the following reference to adjust your magnetic north direction. [URL accessed June 2016] http://www.yourhome.gov.au/sites/prod.yourhome.gov.au/files/images/PD-Orientation-TrueNorth$03 \mathrm{fmt} . \mathrm{png}$
How does your magnetic north compare to your noonday shadow stick north observations?

## After the geometry:

Once the angles have been measured, other factors which need to be considered nearby local features that will block 'your' sunlight.

List some nearby local features that may block sunlight.
$\qquad$
$\qquad$
$\qquad$

It is possible to calculate how much reduction occurs and to compensate by changing the angle of the solar panel to gather more sunlight later or earlier in the day.
Which do you think is the worst direction for an obstruction for a solar panel?
Explain your reasoning.
$\qquad$
$\qquad$

Many newer panels can be connected together and some point east, some point west to avoid overshadowing from neighbouring obstructions.
Which direction has the least sunlight available, at all times of the year?

## OPEN ENDED INQUIRY

## INTRODUCTION

These activities are designed for use with the STELR Renewable Energy module and equipment. The STELR website and further information including contact details can be accessed at: www.stelr.org.au

## FOCUS QUESTIONS

- If you are putting solar panels on your roof, how do you position them to generate the most electricity?
- Why are solar panels usually mounted on an angle?
- Is the amount of electricity generated by solar panels affected by the seasons?
- Is the amount of electricity generated by solar panels affected by the time of day?
- Are solar panels more effective at different locations on Earth?

This simple outline is provided as a starting point for an unguided exploration of the factors which affect the performance of photovoltaic solar panels.

## INQUIRY QUESTIONS

What is the relationship between the power output of a solar panel and the angle at which the solar rays hit its surface?
What happens to the output of a solar panel when light rays fall on it at different angles as the Sun moves across the sky during the day?

## THE INVESTIGATION

## You will need:

- STELR solar panel
- STELR voltmeter/ammeter
- STELR solar cell alignment circle


## Your task:

Use the solar panel to gather data that shows how the output power of a solar panel varies as it is tilted towards and away from a light source. Use the template provided to measure angles. Plot a graph of power versus angle and use the graph to determine the solar panel position when power is a maximum.

## Analysis

What other factors should be taken into account when placing a solar panel in a fixed position. For example: If solar panels were to be placed on the roof of a house, where would they be placed and at what angle? Use your results to comment on how effective solar panels are during different times of the day, and different times of the year. (note: $0^{\circ}$ position is maximum available sunlight).

## GLOSSARY

| Term | Meaning |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## APPENDIX ONE: INVESTIGATION PLANNER

| What you are investigating | What do you think will happen? Explain why. |
| :--- | :--- |
| What are you going to investigate? | What is the aim of your investigation? |
| What is your hypothesis? |  |
| Designing your experiment | Which variable(s) will you test? |
| What variables might affect the outcome of your <br> investigation? | What |
| How will you make your tests fair? | What observations and measurements will you <br> need to take? |
| What risks might be involved in conducting your <br> investigation? Hence, what safety precautions do <br> you plan to take? | What materials, including equipment will you <br> need? |
| Heliable? |  |

## Your results

How will you record your observations and measurements?

Design any suitable tables and draw them up ready in your journal.
Can you use symbols and a key to avoid repeated writing of your observations? If so, have these ready under the appropriate tables.

What graphs can you draw and what spreadsheets can you design to display your results and to enable you to identify any patterns and relationships?

## Conducting your investigation

Once your teacher has approved your plans and supplied the materials, conduct your investigation. Record how you performed the investigation, in your journal. Be sure to include any modifications you made to your plans and the reasons for them.

## Analysing your results: your conclusions

Examine your results. Use them to answer your aim.

From your conclusions, were your predictions and hypothesis correct? Does your hypothesis need to be modified? Discuss.

## Evaluating the investigation

How reliable do you think your results were?
Discuss.

How could you modify your procedure to make your results more reliable?

If you were given the opportunity, what further investigation would you carry out to build on what you learned from this investigation?

## APPENDIX TWO: SOLAR CELL ALIGNMENT CIRCLE



## APPENDIX THREE: STELR INCLINOMETER TEMPLATE




AUSTRALIAN MATHS AND SCIENCE PARTNERSHIPS PROGRAM
Thank you from our partners
${ }^{*}$ **

* Southern Cross University

